

Name: _____

Consider a two sample test of means (e.g., determining that treatment A extends life - on average - more than treatment B). Answer the questions below with respect to the logic of hypothesis testing.

1. Assume the null hypothesis is true.
 - *What does it mean for the null hypothesis to be true? Explain in words.*
2. Generate a sampling distribution for the relevant test statistic under the null hypothesis.
 - *What is a test statistic? What is THE (standard) test statistic? (provide either the technical formula or the general idea).*
 - *What is a sampling distribution? What is THE (standard) sampling distribution? (provide either the technical formula or the general idea).*
3. Compare the observed statistic to the sampling distribution to get the p-value.
 - *What is the p-value? (provide either the definition or the general idea).*

Solution:

1. Assume the null hypothesis is true.
 - *What does it mean for the null hypothesis to be true?*
IF THE NULL HYPOTHESIS IS TRUE, THE AVERAGE INCREASE IN EXTENDED SURVIVAL FOR TREATMENT A IS THE SAME AS THAT FOR TREATMENT B. THE NOTATION FOR TRUE TREATMENT AVERAGES IS OFTEN GIVEN BY μ , LEADING TO A NULL HYPOTHESIS OF:
2. Generate a sampling distribution for the relevant test statistic under the null hypothesis.
 - *What is a test statistic? What is THE test statistic?*
A TEST STATISTIC IS A NUMBER THAT COMES FROM A SAMPLE AND MEASURES SOMETHING ABOUT THE DATA, USUALLY IN REFERENCE TO THE NULL HYPOTHESIS. HERE, INTEREST IS IN COMPARING THE TREATMENT AVERAGES, SO THEIR DIFFERENCE IS PARAMOUNT. THE T-TEST STATISTIC IS GIVEN BY:

$$t^* = \frac{\bar{Y}_A - \bar{Y}_B}{\sqrt{s_A^2/n_A + s_B^2/n_B}}$$

- *What is a sampling distribution? What is THE sampling distribution?*
A SAMPLING DISTRIBUTION REPRESENTS THE POSSIBLE VALUES AND RELATIVE FREQUENCIES OF THE TEST STATISTIC, TYPICALLY UNDER THE ASSUMPTION THAT THE

NULL HYPOTHESIS IS TRUE. THE SAMPLING DISTRIBUTION GIVES A SENSE OF HOW DIFFERENT \bar{Y}_A AND \bar{Y}_B CAN BE WHEN THE NULL HYPOTHESIS REALLY IS TRUE. THAT IS, THE SAMPLING DISTRIBUTION REPRESENTS THE NATURAL VARIABILITY OF THE TEST STATISTIC WHEN H_0 IS TRUE. HERE:

$$t^* \sim t_{df} \quad df \approx \min(n_A - 1, n_B - 1)$$

3. Compare the observed statistic to the sampling distribution to get the p-value.

- *What is the p-value?*

THE P-VALUE IS THE PROBABILITY OF THE OBSERVED DATA OR MORE EXTREME IF H_0 IS TRUE. THAT IS, THE P-VALUE DETERMINES WHERE ON THE SAMPLING DISTRIBUTION OUR OBSERVED TEST STATISTIC IS. IF THE OBSERVED TEST STATISTIC IS NOT CONSISTENT WITH THE SAMPLING DISTRIBUTION (I.E., P-VALUE IS TINY), THEN H_0 CAN BE DISCOUNTED (REJECTED!) AS A POSSIBILITY. IF THE OBSERVED TEST STATISTIC IS CONSISTENT WITH THE SAMPLING DISTRIBUTION (I.E., P-VALUE IS BIG), THEN THERE IS NO REASON TO REJECT H_0 .