# WU \#20 - hierarchical clustering 

Math 154 - Jo Hardin

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Name: $\qquad$
Consider the distances between the following observations:

|  | A | B | C | D | E |
| :--- | :--- | :--- | :--- | :--- | :--- |
| A | 0 |  |  |  |  |
| B | 0.2 | 0 |  |  |  |
| C | 0.6 | 0.5 | 0 |  |  |
| D | 1 | 0.9 | 0.4 | 0 |  |
| E | 0.9 | 0.8 | 0.5 | 0.3 | 0 |

Start with all objects in separate "clusters" (i.e., start with 5 clusters), by merging (complete linkage) one pair of clusters at a time, provide each clustering for $k=5,4,3,2,1$.

## Solution:

$k=4:$ Link A and B to get $-(\mathrm{AB}), \mathrm{C}, \mathrm{D}, \mathrm{E}$
$k=3:$ Link D and E to get $-(\mathrm{AB}), \mathrm{C},(\mathrm{DE})$

$$
\begin{align*}
d_{(A B) C} & =\max \left(d_{A C}, d_{B C}\right)=0.6  \tag{1}\\
d_{(A B) D} & =\max \left(d_{A D}, d_{B D}\right)=1.0  \tag{2}\\
d_{(A B) E} & =\max \left(d_{A E}, d_{B E}\right)=0.9 \tag{3}
\end{align*}
$$

|  | AB | C | D | E |
| :--- | :--- | :--- | :--- | :--- |
| AB | 0 |  |  |  |
| C | 0.6 | 0 |  |  |
| D | 1.0 | 0.4 | 0 |  |
| E | 0.9 | 0.5 | 0.3 | 0 |

Link D and E!
$k=2:$ Link C with (DE) to get $-(\mathrm{AB}),(\mathrm{CDE})$

$$
\begin{align*}
d_{(A B) C} & =0.6  \tag{5}\\
d_{(A B)(D E)} & =\max \left(d_{A D}, d_{B D}, d_{A E}, d_{B E}\right)=1.0  \tag{6}\\
d_{(D E) C} & =\max \left(d_{C D}, d_{C E}\right)=0.5 \tag{7}
\end{align*}
$$

|  | AB | C | DE |
| :--- | :--- | :--- | :--- |
| AB | 0 |  |  |
| C | 0.6 | 0 |  |
| DE | 1.0 | 0.5 | 0 |

Link C with (DE)!
$k=1$ : Link all to get $-(\mathrm{ABCDE})$

$$
\begin{equation*}
d_{(A B)(C D E)}=d_{A D}=1 \tag{9}
\end{equation*}
$$

