## WS #8 - Permuting

## Monday, September 29, 2025

Math 154 - Jo Hardin

Name:
Names of people you worked with:
What is your favorite 5C dining hall? Why?
Γask:
Consider the multiple linear regression model:
$E[Y] = \beta_{0\cdot 1\cdot 2} + \beta_{1\cdot 2}X_1 + \beta_{2\cdot 1}X_2$

[Note:  $\beta_{1\cdot 2}$  means "the coefficient on  $X_1$  given that  $X_2$  is in the model."]

Your friend proposes to permute the variable  $X_1$  in order to create a null sampling distribution for  $b_{1\cdot 2}$  (that is, the distribution of  $b_{1\cdot 2}$  under the condition that the null hypothesis,  $H_0:\beta_{1\cdot 2}=0$ , is true).

In what way does your friend's permutation scheme violate the exchangeability condition?

## Solution:

In the original dataset, there may be a relationship between  $X_1$  and  $X_2$ . However, in *every* sample where  $X_1$  is permuted, there will be no correlation between  $X_1$  and  $X_2$ .

While permuting  $X_1$  does set up conditions for the null hypothesis to be true ( $\beta_{1\cdot 2} = 0$ ), it also violates exchangeability because the permuted samples come from a different data distribution than the original dataset.