WS #5 - Hypothesis Testing

Monday, September 15, 2025

Math 154 - Jo Hardin

Name: ____

Names of people you worked with: ____

What is the most challenging thing you have to do in the next 10 days?

Task: Consider a two sample test of means (e.g., determining that treatment A extends life - on average - more than treatment B). Answer the questions below with respect to the logic of hypothesis testing.

1. Assume the null hypothesis is true.
   - *What does it mean for the null hypothesis to be true? Explain in words.*
2. Generate a sampling distribution for the relevant test statistic under the null hypothesis.
   - *What is a test statistic? What is THE (standard) test statistic? (provide either the technical formula or the general idea).*
   - *What is a sampling distribution? What is THE (standard) sampling distribution? (provide either the technical details or the general idea).*
3. Compare the observed statistic to the sampling distribution to get the p-value.
   - *What is the p-value? (provide either the definition or the general idea).*

Solution:

1. Assume the null hypothesis is true.

    - *What does it mean for the null hypothesis to be true?*
    If the null hypothesis is true, the **average** increase in extended survival for treatment A is the same as that for treatment B. The notation for true treatment averages is often given by μ, leading to a null hypothesis of:

    $$H_0 : \mu_A = \mu_B$$

2. Generate a sampling distribution for the relevant test statistic under the null hypothesis.

    - *What is a test statistic? What is THE test statistic?*
    A test statistic is a number that **comes from a sample/data** and measures something about the data, usually in reference to the null hypothesis. Here, interest is in comparing the treatment averages, so their difference is paramount. The t-test statistic is given by:
    $$t^* = \frac{\overline{X}_A - \overline{X}_B}{\sqrt{s_A^2/n_A + s_B^2/n_B}}$$

    - *What is a sampling distribution? What is THE sampling distribution?*
    A sampling distribution represents the possible values and relative frequencies of the test statistic, typically under the condition that the null hypothesis is true. The sampling distribution gives a sense of how different $\overline{X}_A$ and $\overline{X}_B$ can be when the null hypothesis really is true. That is, the sampling distribution represents the natural variability of the test statistic when H_0 is true. Here:

    $$t^* \sim t_{df} \quad df \approx \min(n_A - 1, n_B - 1)$$

3. Compare the observed statistic to the sampling distribution to get the p-value.

    - *What is the p-value?*
    The p-value is the probability of the observed data or more extreme if H_0 is true. That is, the p-value determines where on the sampling distribution our observed test statistic is. If the observed test statistic is not consistent with the sampling distribution (i.e., p-value is tiny), then H_0 can be discounted (rejected!) as a possibility. If the observed test statistic is consistent with the sampling distribution (i.e., p-value is big), then there is no reason to reject H_0.