WS #16 - Support Vector Machines 2

Wednesday, November 13, 2024

Your Name: ____

Names of people you worked with: ____

What is something you want to do, but you know it is hard, so you haven't tried (yet)?

Task:

Consider the following 5 data points in R^2.

1. Draw the points in R^2 and argue (1 sentence) that the values are not linearly separable.
2. Transform the points using the function $\phi : R^2 \to R^2$. Plot the data again, this time using the first and second coordinate of the transformed data.

$$\phi(x_1, x_2) = (x_1^2, x_2)$$

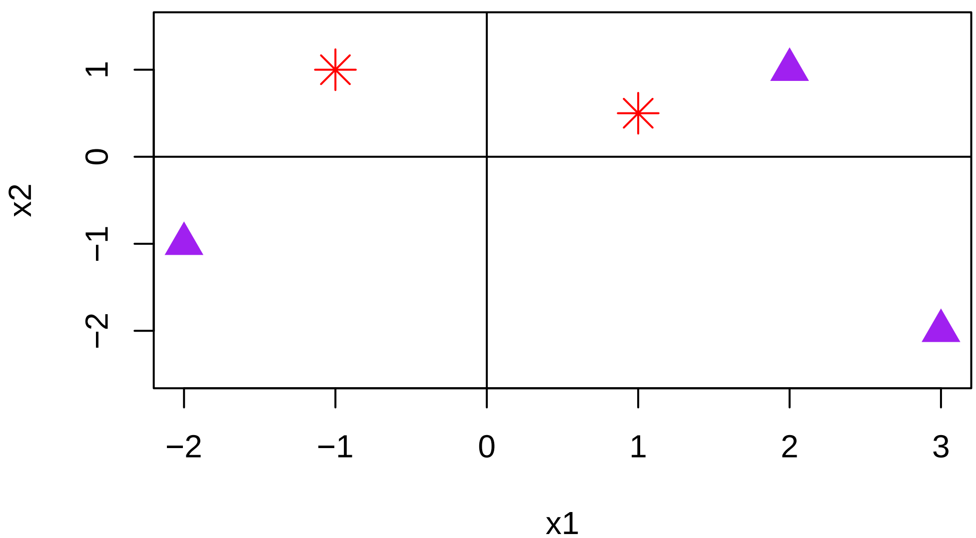
3. Sketch the optimal separating hyperplane and the maximal margin on the second figure (that is, in the transformed space: (ϕ_1, ϕ_2)).
4. Sketch the separating boundary (which will not necessarily be linear) back into the original space: (x_1, x_2).

point	x1	x2	class
1	-1	1	red
2	1	0.5	red
3	2	1	purple
4	-2	-1	purple
5	3	-2	purple

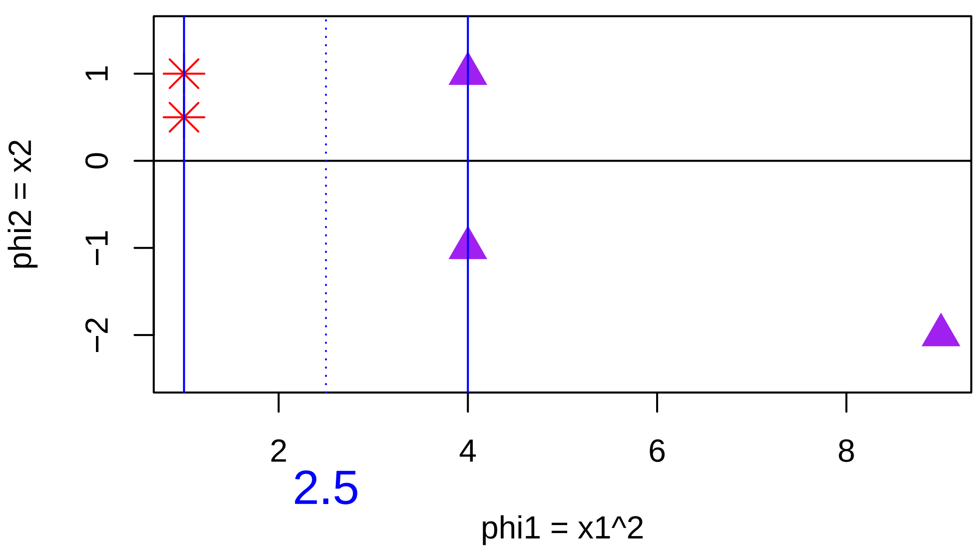
Solution:

There are red and purple points on both sides of any two-dimensional hyperplane (i.e., line) one could draw. Therefore, the points are not linearly separable.

In the original space:



In the transformed space with the separating hyperplane:



Note that the $\mathbf{w}$ vector here is the one such that $\mathbf{w} \cdot \mathbf{u} + b \geq 0$ assigns to purple (in the transformed space). Therefore,

$$
\begin{aligned}
x1^2 &\geq 2.5 \\
\iff x1^2 - 2.5 &\geq 0 \\
\iff x1^2 \times 1 + x2 \times 0 - 2.5 &\geq 0 \\
\iff (x1^2, x2) \cdot (1, 0) - 2.5 &\geq 0 \\
\iff (\phi 1, \phi 2) \cdot (1, 0) - 2.5 &\geq 0
\end{aligned}
$$

So $\mathbf{w} = (1, 0)$ and $b = -2.5$ in the transformed ϕ space

Back in the original space with the decision boundary

In the original space, there are two "lines" which create the prediction boundary. It should make sense that a linear separating hyperplane in ϕ space does not necessarily produce a linear separating hyperplane in the original space.